

# Stability of a $d$ -dimensional thin-shell wormhole surrounded by quintessence

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We present a solution of thin-shell wormholes through cut and paste technique in  $d$ -dimensional general relativity surrounded by quintessence in  $d$ -dimensions. The speciality of the solution is the asymptotic structures are determined by the state of the quintessential matter and have different throat geometries, namely, spherical, planar and hyperbolic. For the construction thin-shell, we use the Darmois-Israel formalism to formulate the stresses, which are concentrated at the throat of the wormhole, are determined. We explore the stability of the thin-shell wormhole solutions by considering linearized radial perturbations around static solutions. We also recovered the previous results from a general solution of stability and obtained a new extra stable solution for some suitable choices of the parameters.

PACS numbers: 04.50.-h, 04.50.Kd, 04.20.Jb

## I. INTRODUCTION

One of the most interesting solutions in General Relativity (GR) are known as wormhole solutions. GR predicts that the spacetime is deformable due to energy/matter and hence it allows the existence of this exotic geometries. Basically, a wormhole is a tunnel which connects two different asymptotically flat regions in the spacetime. The first ideas related to wormhole geometries were suggested by Flamm [1] and later by Einstein and Rosen [2]. The later introduced the so-called ‘Einstein-Rosen’ bridge constructed by the Schwarzschild solution by using the idea of two black holes connecting two different regions of spacetime. However, it was demonstrated that this geometry cannot be traversable due to the singularity. In 1962, Fuller and Wheeler [3], employed Kruskal coordinates to describe the geometry of the Schwarzschild wormhole, and showed that it is non traversable. They showed that if such a wormhole somehow opened, it would close up again before even a single photon could be transmitted through it. Interest in the wormholes was stimulated after the influential study of traversable wormholes by Morris & Thorne [4], which was first introduced as a tool for teaching general relativity, as well as for young students to attract into this field. Such geometries act as tunnels from one region of spacetime to another are necessarily associated with violation of energy conditions [4–7], namely exotic matter, within the framework of general relativity. Being a problematic issue, One can restrict such violation to an infinitesimally small thin shell if one adopts Visser’s approach towards wormhole construction [8, 9], even if we can not ignore it completely, is to concentrate it on a thin-shell. Thus, using the cut-and-paste technique one may concentrate the exotic matter at the wormhole throat, i.e., a thin shell wormhole solution. The surface stress-energy tensor components, at the throat, are determined invoking the Darmois-Israel formalism [10, 11], which leads to the Lanczos equations [12–14] and if the equation of state for the matter on the shell is provided we can obtain the dynamical evolution of the wormhole by using solution of the Lanczos equations.

In this scenario, Stability analysis of thin-shell wormholes under linear perturbations preserving the original symmetries have been carried out by several authors. Poisson and Visser [15] performed stability analysis for the Schwarzschild thin-shell wormholes without assuming specific equations of state (EoS) for the exotic matter. Considering the same analysis a number of papers have been extended include charge and cosmological constant, as well as other features (see, for example, references [16–19, 21–27, 56]). A well studied for thin-shell wormholes employing different equation of state have been discussed in [28–34].

The study of objects in higher dimensional spacetimes, which attracted the attention of researchers because of the existence of extra dimension is predicted by the string theory/ M-Theory at low energies, which is described by higher-dimensional theories of gravity, namely various types of supergravities. In the year 1920 Kaluza and Klein unified electromagnetism and gravity by introducing a 5th dimension, while today string theory, as a promising candidate for the quantum theory of gravity and for the unification of all black object in pure gravity are often intimately

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related to black hole/brane solutions. According to the Kaluza–Klein picture, extra dimensions are compactified to a small radius at the order of Planck length, insofar as low energies are concerned. Alternatively, in Braneworld scenarios [35–38], the Standard Model particles were confined to a three-brane, where only gravity is allowed to penetrate the extra dimensions. In this context the 4-dimensional universe is viewed as a hypersurface called the brane, which is embedded in a higher dimensional spacetime called the bulk where all the matter fields are confined to the brane by a variety of different mechanisms, while gravity can propagate in the bulk. During the last two decades, a number of important solutions of Einstein equations in higher dimensions have been performed taking into account the possibility of extra dimensions (for example see [39–45]) for wider understanding of gravitational fields.

Studying the higher dimensional relativity gives the additional motivation to the people by hoping to find possible factors and interesting features are revealed in dimensions other than four which is now extended to the study of wormhole and thin-shell wormhole physics. There are many interesting studies of Lorentzian as well as Euclidean wormholes in higher dimensional gravity. Euclidean wormholes have been studied by Jianjun and Sicong and by Gonzales–Diaz and by [46, 47]. The selfdual Lorentzian wormholes have been studied in the context of the  $N$ -dimensional Einstein gravity by Mauricio Cataldo et. al [48]. They have also studied the  $(N+1)$ -dimensional evolving wormholes supported by energy satisfying a polytropic equation of state [49]. Higher dimensional evolving wormholes have been studied in [50–52] and Einstein–Gauss–Bonnet theory of gravitation [54, 61]. Electrically charged thin-shell wormholes in higher dimensional gravity with a cosmological constant have been discussed in [55, 56].  $d$ -dimensional non-asymptotically flat thin-shell wormholes in Einstein–Yang–Mills–dilaton (EYMD) gravity were considered in [57] and its extension to Einstein–Yang–Mills–Gauss–Bonnet is given in [57, 58]. Recently a well studied higher dimensional thin-shell wormholes have been found in [59, 60]

This article is organized as follows: Sec. II establishes the basic notions for a  $d$ -dimensional spherically symmetric space-time surrounded by quintessence. In Sec. III, we construct thin-shell wormholes for this space-time by using the cut-and-paste technique. In addition, we directly derive the geodesic equation for a test particle which moves radially and initially is at rest. Sec. IV is devoted to study the stability analysis using the standard linearised expansion method. Within this sections, we derive the general stability conditions for the thin shell wormhole. In Sec. V we demonstrate that from our general result, some interesting different particular cases studied before can be recovered. Additionally, we explore one new example and study the stability regions depending on the parameters. Finally, in Sec. VI we conclude our main results.

## II. $d$ -DIMENSIONAL SPHERICALLY SYMMETRIC SPACETIME SURROUNDED BY QUINTESSENCE

Let us consider a  $d$ -dimensional static spherically symmetric metric surrounded by quintessence, which reads [62]

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega_{d-2}^2. \quad (1)$$

where  $\Omega_{d-2}^2$  represents the metric of the  $(d-2)$  unit-sphere and the metric function is given by

$$f(r) = \left[ 1 - \frac{2M}{r^{d-3}} - \frac{C}{r^{(d-1)w_q+d-3}} \right]. \quad (2)$$

The space-time depends on the dimension  $d$ , the quintessence state parameter  $w_q \leq 0$ , mass  $M$  and a constant  $C$ . Moreover, we also note that the spacetime is asymptotically flat if  $0 > w_q > -(d-3)/(d-1)$  and is asymptotically dS-like when  $-1 \leq w_q < -(d-3)/(d-1)$ . For instance, when  $w_q = -1$ , the metric Eq. (1) becomes

$$ds^2 = - \left[ 1 - \frac{2M}{r^{d-3}} - Cr^2 \right] dt^2 + \left[ 1 - \frac{2M}{r^{d-3}} - Cr^2 \right]^{-1} dr^2 + r^2 d\Omega_{d-2}^2, \quad (3)$$

which reduces to the  $d$ -dimensional Schwarzschild de Sitter black hole where  $C$  is the cosmological constant. Moreover, the metric Eq. (1) reduces to the  $d$ -dimensional Reissner–Nordström black hole if the quintessence state parameter takes the following form

$$w_q = \frac{d-3}{d-1}. \quad (4)$$

In the following section, we will focus our study on the construction of thin-shell wormholes in this spacetime.

### III. CONSTRUCTION OF THIN-SHELL WORMHOLE AND THE GRAVITATIONAL FIELD

We start the mathematical construction for our thin-shell wormhole considering two identical copies of the vacuum solution and removing from each copy the spacetime region given by

$$\Omega^\pm \equiv \{r^\pm \leq a | a > r_h\}, \quad (5)$$

where  $a$  is a radius constant greater than the event horizon  $r_h$  to avoid the presence of horizon and singularities for the metric (1). The removal of the regions from each spacetime gives two geodesically incomplete manifolds, with the timelike hypersurfaces as boundaries given by

$$\partial\Omega^\pm \equiv \{r^\pm = a | a > r_h\}. \quad (6)$$

We obtain a geodesically complete manifold by identifying the timelike hyperurface  $\partial\Omega^+ = \partial\Omega^-$ , where two regions are connected by a wormhole. The identified region  $\partial\Omega$  is called the “throat of the wormhole” where the exotic matter is concentrated. The induced metric on the hypersurface  $\partial\Omega$  takes the following form

$$ds^2 = -d\tau^2 + a^2(\tau)d\Omega_{d-2}^2, \quad (7)$$

where  $\tau$  is the proper time along the hypersurface  $\partial\Omega$  and  $a(\tau)$  defines the radius of the throat as a function of the proper time. The surface stress at the junction boundary are determined using the Darmois-Israel formulation.

The Lanczos equation gives the intrinsic surface stress-energy tensor  $S_{ij}$  which reads,

$$S_j^i = -\frac{1}{8\pi} (\kappa_j^i - \delta_j^i \kappa_m^m), \quad (8)$$

where the quantity  $k_{ij} = K_{ij}^+ - K_{ij}^-$  represents the second fundamental form of the extrinsic curvature  $K_{ij}^\pm$ . Note that the symbol  $-$  and  $+$  corresponds to the interior and exterior spacetime respectively. The second fundamental form the extrinsic curvature can be defined as follows

$$K_{ij}^\pm = -\eta_\nu \left( \frac{\partial^2 x^\nu}{\partial \xi^i \partial \xi^j} + \Gamma_{\alpha\beta}^{\nu\pm} \frac{\partial x^\alpha}{\partial \xi^i} \frac{\partial x^\beta}{\partial \xi^j} \right), \quad (9)$$

where  $\eta_\nu$  represents the unit normal vector at the junction and  $\xi^i$  represents the intrinsic co-ordinates. At the hypersurface  $\partial\Omega$ , the parametric equation is given by  $f(x^\mu(\xi^i)) = 0$ . By using this equation, we derive the formula for the unit normal vector to the hypersurface  $\partial\Omega$ , which is given by

$$n_\mu = \pm \left| g^{\alpha\beta} \frac{\partial f}{\partial x^\alpha} \frac{\partial f}{\partial x^\beta} \right|^{-\frac{1}{2}} \frac{\partial f}{\partial x^\mu}, \quad (10)$$

where the unitary condition  $n_\mu n^\mu = +1$  holds and the discontinuity of the extrinsic curvature  $\kappa_{ij}$  can be written in a simplified form due to spherical symmetry as  $\kappa_j^i = \text{diag}(\kappa_\tau^\tau, \kappa_{\theta_1}^{\theta_1}, \dots, \kappa_{\theta_{d-2}}^{\theta_{d-2}})$ . Therefore, we can write the surface-energy tensor as  $S_j^i = \text{diag}(-\sigma, P, \dots, P)$ , where  $\sigma$  is the surface energy density and  $P$  is the surface pressure. Now, by using the Lanczos equation we find that

$$\sigma(a) = -\frac{(d-2)}{4\pi a} \sqrt{f(a) + \dot{a}^2}, \quad (11)$$

and

$$P(a) = -\frac{d-3}{d-2} \sigma + \frac{f'(a) + \ddot{a}}{8\pi \sqrt{f(a) + \dot{a}^2}}, \quad (12)$$

where primes and dots denote differentiation with respect to  $a$  and  $\tau$  respectively and the function  $f(a)$  is given in Eq. (2). Here, the energy density  $\sigma$  and surface pressure  $P$  obey the following conservation equation

$$\frac{d}{d\tau} (\sigma a^{d-2}) + P \frac{d}{d\tau} (a^{d-2}) = 0. \quad (13)$$

For a static configuration of radius  $a = a_0$ , we have  $\dot{a} = 0$  and  $\ddot{a} = 0$  so that, from Eqs. (11) and (12) we directly find

$$\sigma(a_0) = -\frac{(d-2)}{4\pi a_0} \sqrt{f(a_0)}, \quad (14)$$

and

$$P(a_0) = -\frac{d-3}{d-2}\sigma + \frac{f'(a_0)}{8\pi\sqrt{f(a_0)}}. \quad (15)$$

Let us now analyse the attraction and repulsive nature of the wormhole on test particles, so we calculate the four-acceleration for the static wormhole ( $\dot{a} = 0$ ), which can be written as

$$a^\mu = u^\mu_{;\nu} u^\nu, \quad (16)$$

where the 4-velocity is  $u^\mu = \frac{dx^\mu}{d\tau} = (\frac{1}{\sqrt{f(r)}}, 0, 0, \dots, 0)$ . The only non-zero component of the acceleration is given by

$$a^r = \Gamma^r_{tt} \left( \frac{dt}{d\tau} \right)^2 = \frac{M(d-3)}{r^{d-2}} + \frac{C[(d-1)w_q + d-3]}{2r^{(d-1)w_q + d-2}}. \quad (17)$$

Let us now consider a test particle which moves in radial direction and initially it is at rest. The equation of motion for this particle takes the following form

$$\frac{d^2 r}{d\tau^2} = -\Gamma^r_{tt} \left( \frac{dt}{d\tau} \right)^2 = -a^r, \quad (18)$$

which gives the geodesic equation if  $a^r = 0$ . From here we can notice that the wormhole is attractive if  $a^r > 0$  and repulsive if  $a^r < 0$ .

#### IV. LINEARIZED STABILITY ANALYSIS

Equations of motion (11) and (12) can be rewritten in the following form

$$\dot{a}^2 - \frac{16\pi^2 a^2}{(d-2)^2} \sigma^2 = -1 + \frac{2M}{a^{d-3}} + \frac{C}{a^{(d-1)w_q + d-3}}, \quad (19)$$

$$\dot{\sigma} = -(d-2) \frac{\dot{a}}{a} (\sigma + P). \quad (20)$$

Now, if we integrate the energy conservation equation (13), we get

$$\ln(a) = -\frac{1}{d-2} \int \frac{d\sigma}{\sigma + P(\sigma)}, \quad (21)$$

which can be formally inverted to provide  $\sigma = \sigma(a)$ . To find the stability conditions for our configuration, we consider linear perturbations around a static solution with radius  $a_0$  [15]. The surface energy density  $\sigma(a_0)$  and the surface pressure  $P(a_0)$  for the static solution are explicitly given in Eqs. (14) and (15) respectively. Now, the thin shell equation of motion can be obtained by rewriting Eq. (19) as

$$\dot{a} + V(a) = 0, \quad (22)$$

where the potential  $V(a)$  is defined as

$$V(a) = f(a) - \frac{16\pi^2 a^2}{(d-2)^2} \sigma^2. \quad (23)$$

Since we are linearising around the static solution  $a_0$ , we expand  $V(a)$  around  $a_0$  using Taylor series expansion up to second order in powers of  $(a - a_0)$ , which provides us

$$V(a) = V(a_0) + V'(a_0)(a - a_0) + \frac{1}{2} V''(a_0)(a - a_0)^2 + \mathcal{O}[(a - a_0)^3], \quad (24)$$

where prime denotes derivatives with respect to  $a$ . The first order derivative of  $V(a)$  is given by

$$V'(a) = f'(a) - \frac{32\pi^2}{(d-2)^2} [\sigma + a\sigma'] a\sigma. \quad (25)$$

Now, if we use Eq. (20), which is the conservation equation of the surface stress energy tensor, the above expression becomes

$$V'(a) = f'(a) + \frac{32\pi^2}{(d-2)^2} a [(d-3)\sigma^2 + (d-2)\sigma P]. \quad (26)$$

For the second derivative of the potential, we define a parameter  $\eta(\sigma) = dp/d\sigma = P'/\sigma'$ , which physically is interpreted as the subluminal speed of sound. Hence, the second derivative of the potential can be written as

$$V''(a) = f''(a) - \frac{32\pi^2}{(d-2)^2} \left\{ [(d-3)\sigma + (d-2)P]^2 + (d-2)(d-3 + (d-2)\eta)\sigma(\sigma + P) \right\}. \quad (27)$$

Since we are linearising around  $a = a_0$ , we can now go back to Eqs. (23) and (26) to substitute for  $a = a_0$  to find that  $V(a_0) = 0$  and  $V'(a_0) = 0$ , respectively. Therefore, the potential  $V(a)$  from Eq. (24) is reduced to

$$V(a) = \frac{1}{2} V''(a_0)(a - a_0)^2 + \mathcal{O}[(a - a_0)^3], \quad (28)$$

so that the equation of motion of the wormhole throat is given by

$$\dot{a}^2 = -\frac{1}{2} V''(a_0)(a - a_0)^2 + \mathcal{O}[(a - a_0)^3]. \quad (29)$$

Thus, the wormhole is stable if and only if  $V''(a_0) > 0$ . Hence,  $V(a_0)$  has a local minimum at  $a_0$ . To carry out this analysis, we can study which conditions we need for having stable wormholes. In our case, we find that this parameter needs to satisfy

$$\eta_0 < \frac{a_0^2 f_0'^2 - 2a_0^2 f_0'' f_0}{2(d-2)f_0(a_0 f_0' - 2f_0)} - \frac{d-3}{d-2}, \quad (30)$$

where all the quantities with a suffix 0 denote that they are evaluated at  $a = a_0$ .

Since we are interested in a more general scenario, we have to take into account the relation between the ADM mass  $\mathcal{M}$  and the black hole mass parameter  $M$  in the case of a  $d$ -dimensional spacetime. More specifically, in the case of a spherical geometry ( $k = 1$ ), we have the following relation

$$\mathcal{M} = \frac{16\pi\Gamma(\frac{d-1}{2})}{(d-2)2\pi^{\frac{d-1}{2}}} M. \quad (31)$$

Therefore, Eq. (30) can also be written as

$$\eta_0 < \frac{a_0^2 f_0'^2 - 2a_0^2 f_0'' f_0}{2(d-2)f_0(-2f_0 + \frac{\mathcal{M}(d-3)}{a_0^{d-3}} + \frac{C[(d-1)w_q + d-3]}{a_0^{(d-1)w_q + d-3}})} - \frac{d-3}{d-2}, \quad \text{if } -2f_0 + \frac{\mathcal{M}(d-3)}{a_0^{d-3}} + \frac{C[(d-1)w_q + d-3]}{a_0^{(d-1)w_q + d-3}} > 0 \quad (32)$$

and

$$\eta_0 > \frac{a_0^2 f_0'^2 - 2a_0^2 f_0'' f_0}{2(d-2)f_0(-2f_0 + \frac{\mathcal{M}(d-3)}{a_0^{d-3}} + \frac{C[(d-1)w_q + d-3]}{a_0^{(d-1)w_q + d-3}})} - \frac{d-3}{d-2}, \quad \text{if } -2f_0 + \frac{\mathcal{M}(d-3)}{a_0^{d-3}} + \frac{C[(d-1)w_q + d-3]}{a_0^{(d-1)w_q + d-3}} < 0. \quad (33)$$

These two above inequalities gives us two set of conditions where the wormhole is stable or not depending on all the parameters of the space-time. Following Dias and Lemos approach [55], we can write the general static metric solution for a  $d$ -dimensional solution with different  $(d-2)$  geometric-topologies which are encoded by the geometric-topological factor  $k$ . In that case, the function  $f_0 \equiv f(a_0)$ , given in Eq. (2) evaluated at  $a_0$  is given by

$$f(a_0) = k - \frac{\mathcal{M}}{a_0^{d-3}} - \frac{C}{a_0^{(d-1)w_q + d-3}}, \quad (34)$$

where we have three special cases:  $k = 1$  for spherical,  $k = 0$  for planar, and  $k = -1$  for hyperbolic geometries. Moreover for  $f_0'$  and  $f_0''$  we have

$$f'(a_0) = \frac{\mathcal{M}(d-3)}{a_0^{d-2}} + \frac{C[(d-1)w_q + d-3]}{a_0^{(d-1)w_q + d-2}}, \quad (35)$$

and

$$f''(a_0) = -\frac{\mathcal{M}(d-3)(d-2)}{a_0^{d-1}} - \frac{C[(d-1)w_q + d-3][(d-1)w_q + d-2]}{a_0^{(d-1)w_q + d-1}}. \quad (36)$$

## V. SPECIAL CASES AND NEW EXAMPLE

We now turn our attention for some special cases which were already studied by some authors. We will see that from our general result, we can recover the same stability conditions for a particular set of choices for the parameters. Additionally, we also study further a new interesting example coming from our spacetime.

### A. Poisson–Visser, $d = 4$ , $k = 1$ , $M \neq 0$ , $C = 0$ and $w_q = 0$

Let us start with the simplest wormhole stability solution, namely, the four-dimensional solution with spherical symmetry without charge and quintessence. In other words by setting  $d = 4$ ,  $k = 1$ ,  $M \neq 0$ ,  $C = 0$  and  $w_q = 0$  from Eq. (32) and (33) we find that the stability conditions become

$$\eta_0 < \frac{-1 + 3M/a_0 - 3M^2/a_0^2}{2 \left(1 - \frac{2M}{a_0}\right) \left(1 - \frac{3M}{a_0}\right)}, \quad \text{if } 1 - \frac{3M}{a_0} > 0 \quad (37)$$

and

$$\eta_0 > \frac{-1 + 3M/a_0 - 3M^2/a_0^2}{2 \left(1 - \frac{2M}{a_0}\right) \left(1 - \frac{3M}{a_0}\right)}, \quad \text{if } 1 - \frac{3M}{a_0} < 0. \quad (38)$$

Note that in  $d = 4$ , we have used the fact that  $\mathcal{M} = 2M$  since  $\Gamma(3/2) = \sqrt{\pi}/2$ . The above result first was found by Poisson and Visser in [15].

### B. Eiroa–Romero, $d = 4$ , $k = 1$ , $M \neq 0$ , $C \neq 0$ and $w_q = 1/3$

Our solution can be extended to study the stability of four-dimensional spherical symmetry charged thin shell wormhole, which corresponds to  $d = 4$  and  $w_q = 1/3$ . From Eq. (32) and (33) it follows that this wormhole will be stable if

$$\eta_0 < \frac{-1 + 3M/a_0 - 3M^2/a_0^2 - MC/a_0^3}{2 \left(1 - \frac{2M}{a_0} - \frac{C}{a_0^2}\right) \left(1 - \frac{3M}{a_0} - \frac{2C}{a_0^2}\right)}, \quad \text{if } 1 - \frac{3M}{a_0} - \frac{2C}{a_0^2} > 0 \quad (39)$$

and

$$\eta_0 > \frac{-1 + 3M/a_0 - 3M^2/a_0^2 - MC/a_0^3}{2 \left(1 - \frac{2M}{a_0} - \frac{C}{a_0^2}\right) \left(1 - \frac{3M}{a_0} - \frac{2C}{a_0^2}\right)}, \quad \text{if } 1 - \frac{3M}{a_0} - \frac{2C}{a_0^2} < 0 \quad (40)$$

Furthermore, if we replace  $C = -Q^2$  in the above equations we recover the Reissner–Nordstrom TSW solution reported by Eiroa and Romero in [16].

### C. Lobo–Crawford, $d = 4$ , $k = 1$ , $M \neq 0$ , $C \neq 0$ and $w_q = -1$

We will now show that from Eqs (32) and (33) one can recover a spherically symmetric four-dimensional TSW solution with a cosmological constant. This case can be recovered by setting the quintessence parameter  $w_q = -1$  and  $d = 4$ . From Eqs (32) and (33) is not difficult to show that if

$$\eta_0 < \frac{-1 + 3M/a_0 - 3M^2/a_0^2 + 3C M a_0}{2 \left(1 - \frac{3M}{a_0}\right) \left(1 - \frac{2M}{a_0} - C a_0^2\right)}, \quad \text{if } a_0 > 3M \quad (41)$$

and

$$\eta_0 > \frac{-1 + 3M/a_0 - 3M^2/a_0^2 + 3C M a_0}{2 \left(1 - \frac{3M}{a_0}\right) \left(1 - \frac{2M}{a_0} - C a_0^2\right)}, \quad \text{if } a_0 < 3M, \quad (42)$$

the wormhole will be stable. Note that if we choose  $C = \Lambda/3$ , these inequalities are reduced to the solution found by Lobo and Crawford in [17].

**D. Rahaman–Kalam–Chakraborty,  $d = d$ ,  $k = 1$ ,  $M \neq 0$  and  $w_q = (d-3)/(d-1)$**

Further interesting generalizations can be recovered if we consider a  $d$ -dimensional spacetime with spherical geometry with the quintessence parameter being  $w_q = (d-3)/(d-1)$ . Note that in this case we have a charged  $d$ -dimensional TSW with spherical geometry. Here we also need to write the relation between the charge parameter  $Q$  and the ADM charge  $\mathcal{Q}$  which is given by

$$\mathcal{Q}^2 = \frac{2}{(d-2)(d-3)} Q^2. \quad (43)$$

Moreover, by setting  $C = -\mathcal{Q}^2$  and by using the general conditions (32) and (33), it follows that the wormhole will be stable if

$$\eta_0 < \frac{a_0^2 f_0'^2 - 2a_0^2 f_0'' f_0}{2(d-2)f_0(-2 + \frac{\mathcal{M}(d-1)}{a_0^{d-3}} - \frac{2\mathcal{Q}^2(d-2)}{a_0^{2(d-3)}})} - \frac{d-3}{d-2}, \quad \text{if } -2 + \frac{\mathcal{M}(d-1)}{a_0^{d-3}} - \frac{2\mathcal{Q}^2(d-2)}{a_0^{2(d-3)}} > 0 \quad (44)$$

and

$$\eta_0 > \frac{a_0^2 f_0'^2 - 2a_0^2 f_0'' f_0}{2(d-2)f_0(-2 + \frac{\mathcal{M}(d-1)}{a_0^{d-3}} - \frac{2\mathcal{Q}^2(d-2)}{a_0^{2(d-3)}})} - \frac{d-3}{d-2}, \quad \text{if } -2 + \frac{\mathcal{M}(d-1)}{a_0^{d-3}} - \frac{2\mathcal{Q}^2(d-2)}{a_0^{2(d-3)}} < 0. \quad (45)$$

This solution in the literature was first studied by Rahaman–Kalam–Chakraborty [56].

**E. Dias–Lemos,  $d = d$ ,  $k = 1, 0, -1$ ,  $M \neq 0$  and  $w_q = -1$**

Let us now recover the solution to a  $d$ -dimensional TSW with a cosmological constant and vanishing charge, i.e.  $Q = 0$ . This result can be found from Eqs. (32) and (33) by setting the quintessence parameter  $w_q = -1$ . After some algebraic manipulation we find that the stability conditions become

$$\eta_0 < \frac{a_0^2 f_0'^2 - 2a_0^2 f_0'' f_0}{2(d-2)f_0(-2k + \frac{\mathcal{M}(d-1)}{a_0^{d-3}})} - \frac{d-3}{d-2}, \quad \text{if } -2k + \frac{\mathcal{M}(d-1)}{a_0^{d-3}} > 0 \quad (46)$$

and

$$\eta_0 > \frac{a_0^2 f_0'^2 - 2a_0^2 f_0'' f_0}{2(d-2)f_0(-2k + \frac{\mathcal{M}(d-1)}{a_0^{d-3}})} - \frac{d-3}{d-2}, \quad \text{if } -2k + \frac{\mathcal{M}(d-1)}{a_0^{d-3}} < 0. \quad (47)$$

The  $d$ -dimensional TSW with a cosmological constant and three different geometries encoded by the parameter  $k$  was recently investigated by Dias and Lemos. If we let  $w_q = -1$  and  $C = \Lambda/3$ , the function  $f_0 = f(a_0)$  can be written as

$$f(a_0) = k - \frac{\mathcal{M}}{a_0^{d-3}} - \frac{\Lambda a_0^2}{3}, \quad (48)$$

which corresponds to Dias and Lemos solution found recently in [55].

**F. A new example,  $d = d$ ,  $k = 1, 0, -1$ ,  $M \neq 0$  and  $w_q \neq 0$**

Finally we can now consider a more general scenario, namely, a  $d$ -dimensional TSW wormhole surrounded by quintessence  $w_q \neq 0$ . Note that since the energy density  $\rho_q$  for quintessence should be positive and explicitly given as [62]

$$\rho_q = -\frac{C w_q (d-1)(d-2)}{4r^{(d-1)(w_q+1)}}, \quad (49)$$

the quintessence parameter  $w_q$ , must be negative, i.e.  $w_q \leq 0$ . Hence, for this case, the stability conditions Eqs (32) and (33) related to three different geometries become

$$\eta_0 < \frac{a_0^2 f_0'^2 - 2a_0^2 f_0'' f_0}{2(d-2)f_0(-2k + \frac{\mathcal{M}(d-1)}{a_0^{d-3}} + \frac{C(d-1)(w_q+1)}{a_0^{(d-1)w_q+d-3}})} - \frac{d-3}{d-2}, \quad \text{if } -2k + \frac{\mathcal{M}(d-1)}{a_0^{d-3}} + \frac{C(d-1)(w_q+1)}{a_0^{(d-1)w_q+d-3}} > 0, \quad (50)$$

and

$$\eta_0 > \frac{a_0^2 f_0'^2 - 2a_0^2 f_0'' f_0}{2(d-2)f_0(-2k + \frac{\mathcal{M}(d-1)}{a_0^{d-3}} + \frac{C(d-1)(w_q+1)}{a_0^{(d-1)w_q+d-3}})} - \frac{d-3}{d-2}, \quad \text{if} \quad -2k + \frac{\mathcal{M}(d-1)}{a_0^{d-3}} + \frac{C(d-1)(w_q+1)}{a_0^{(d-1)w_q+d-3}} < 0. \quad (51)$$

For more useful informations on the wormhole stability in different geometries and different dimensions we show graphically the wormhole stability in terms of the parameter  $\eta_0$  and  $a_0$ , for different values of  $d$ ,  $\mathcal{M}$ ,  $C$  and  $w_q$ .

In Fig. 1 we show the stability region for the spherical geometry, i.e.  $k = 1$ . As we can see, in the first and second plot there are two interesting intervals that are worth of mentioning. More specifically, the first interval is between the left and the right asymptote with the region of stability above the curve. The second interval is on the right to the right asymptote with the region of stability below the curve. In the third plot there is only one interval worth of mentioning with the region of stability below the curve.

In Fig. 2 we show graphically the stability region for the planar geometry, i.e.  $k = 0$ . There are two important intervals for the first and second plot and only one interval for the third plot. The corresponding region of stability in the first case is above the curve, while in the later case the region of stability is below the curve.

Finally, in Fig. 3 the stability region for the hyperbolic geometry ( $k = -1$ ) is depicted. From the first and second plot we can easily observe that the regions of stability are above (below) the curve in the first (second) interval. Last but not least, we are left with only one interesting interval with the region of stability below the curve in the last plot. From those examples it follows that, the stability domain of the wormhole increases if we increase the number of dimensions.

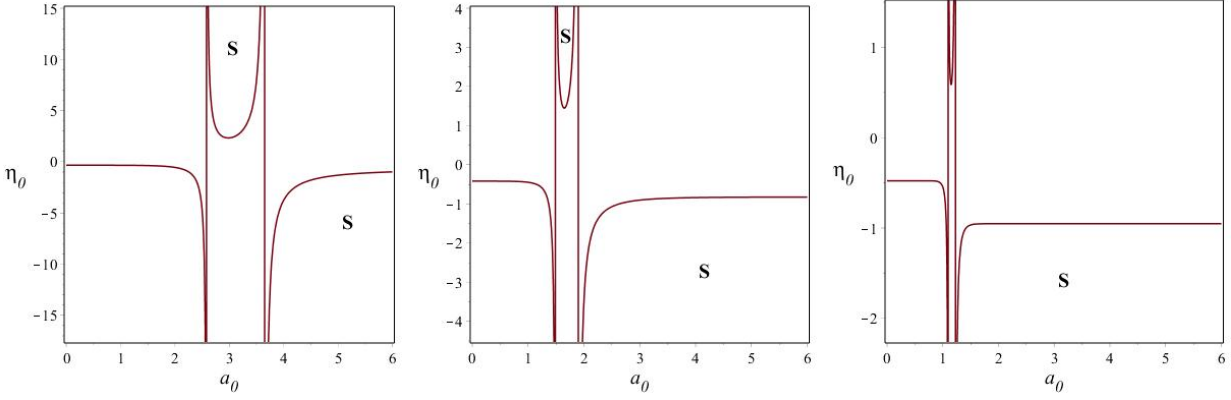


FIG. 1: Stability regions of the wormhole for  $k = 1$  and  $M = 10$  in all cases. In the first case we have chosen  $d = 5$ ,  $C = -0.5$ , and  $w_q = -0.5$ . In the second case  $d = 8$ ,  $C = -0.5$ , and  $w_q = -0.6$  while in the last plot we have chosen  $d = 24$ ,  $C = -0.5$ , and  $w_q = -0.9$ . The stability region is denoted by  $S$ .

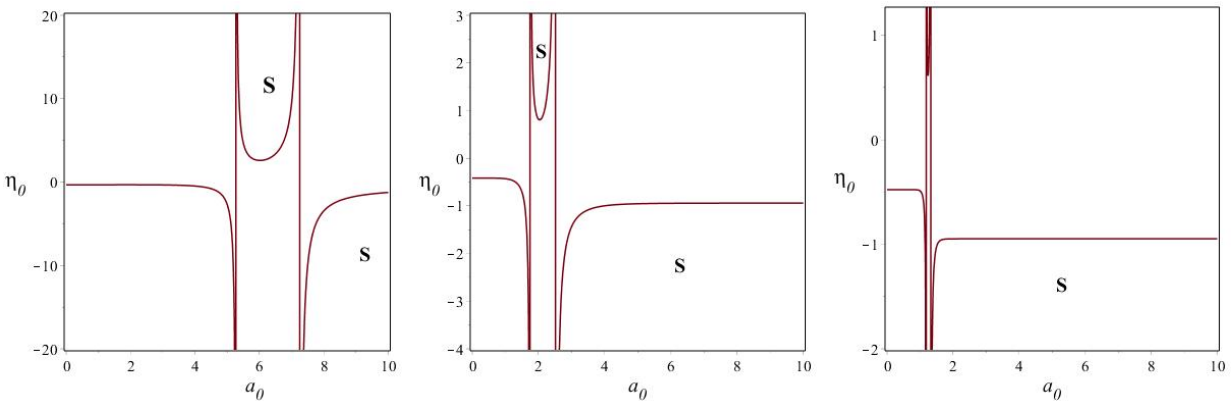


FIG. 2: Stability regions of the wormhole for  $k = 0$  and  $M = 10$  in all cases. In the first case we have chosen  $d = 5$ ,  $C = -0.7$ , and  $w_q = -0.4$ . In the second case  $d = 8$ ,  $C = -0.3$ , and  $w_q = -0.9$  while in the last plot we have chosen  $d = 24$ ,  $C = -0.5$ , and  $w_q = -0.9$ . The stability region is denoted by  $S$ .



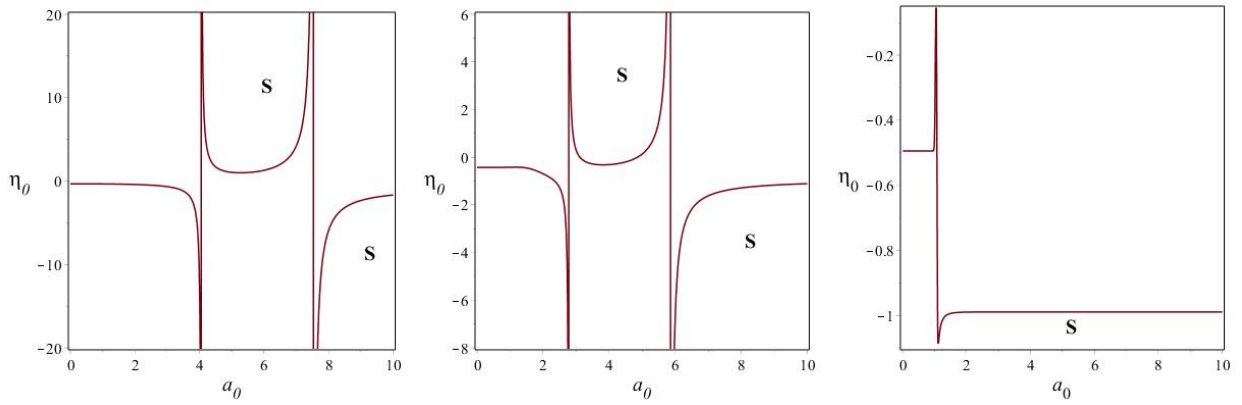


FIG. 3: Stability regions of the wormhole for  $k = -1$  and  $M = 10$  in all cases. In the first plot we have chosen  $d = 5$ ,  $C = -0.3$ , and  $w_q = -0.8$ . In the second plot  $d = 9$ ,  $C = -0.3$ , and  $w_q = -0.9$  while in the third plot we have chosen  $d = 100$ ,  $C = -0.3$ , and  $w_q = -0.9$ . The stability region is denoted by  $S$ .

## VI. CONCLUSIONS

In this work we have used the cut and paste method and the Darmois–Israel formalism to construct a  $d$ -dimensional wormhole surrounded by quintessence. We have obtained a new and a more general solution of a  $d$ -dimensional wormhole and recovered some of the well known wormhole solutions in the literature as special cases of our result. We carry out the stability of the wormhole under a linearised stability analysis in three different geometries and explore the stability regions in different dimensions. In particular we investigate the wormhole stability regions for a spherical geometry, planar geometry, and hyperbolic geometry. Finally we discuss the stability regions in different dimensions  $d$  and investigate the effects of different values of the parameters  $w_q \leq 0$ ,  $\mathcal{M}$ ,  $C$  on wormhole stability. For all three different geometries it is shown that by increasing the number of dimensions, increases the stability domain of obtaining stable  $d$ -dimensional wormholes surrounded by quintessence.

## Acknowledgments

S.B. is supported by the Comisión Nacional de Investigación Científica y Tecnológica (Becas Chile Grant No. 72150066).

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